Chapter 12 Differentiation

0606/12/F/M/19

1. The area of a sector of a circle of radius $r\ cm$ is $36\ cm^2$.
   a. Show that the perimeter, *P* cm, of the sector is such that $P = 2r + \frac{72}{r}$.

   [3]

   b. Hence, given that *r* can vary, find the stationary value of *P* and determine its nature.

   [4]

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2. (a) Given that $y = x\sqrt{x^2 + 1}$, show that $\frac{dy}{dx} = \frac{ax^2+b}{(x^2+1)^p}$, where $a,\ b$ and p are positive constants.

[4]

(b) Explain why the graph of $y = x\sqrt{x^2 + 1}$ has no stationary points.

[2]

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3. It is given that $y = (x^2 + 1)(2x - 3)^{\frac{1}{2}}$.

(i) Show that $\frac{dy}{dx} = \frac{Px^2+Qx+1}{(2x-3)^{\frac{1}{2}}}$, where P and Q are integers.

[5]

(ii) Hence find the equation of the normal to the curve $y = (x^2 + 1)(2x - 3)^{\frac{1}{2}}$ at the point where $x = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[4]

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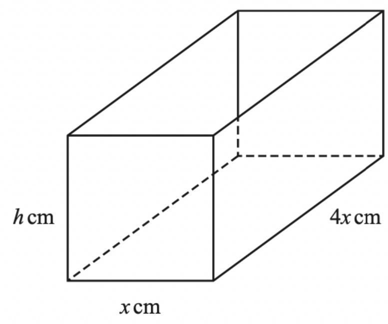
4.



The diagram shows an open container in the shape of a cuboid of width *x* cm, length 4*x* cm and height *h* cm. The volume of the container is $800 cm^3$.

a. Show that the external surface area, $S\ cm^2$, of the open container is such that $S = 4x^2 + \frac{2000}{x}$.

[4]

b. Given that x can vary, find the stationary value of S and determine its nature.

[5]

5. The normal to the curve $y = (x - 2)(3x + 1)^{\frac{2}{3}}$ at the point where $x = \frac{7}{3}$, meets the *y*-axis at the point *P*. Find the exact coordinates of the point *P*.

[7]

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6. .A circle has diameter *x* which is increasing at a constant rate of 0.01 $cm\, s^{-1}$. Find the exact rate of change of the area of the circle when *x* = 6 cm.

[5]

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7. A curve has equation $y = (3x - 5)^2 - 2x$.

a. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

[4]

b. Find the exact value of the *x*-coordinate of each of the stationary points of the curve.

[2]

c. Use the second derivative test to determine the nature of each of the stationary points.

[2]

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8. Find the equation of the normal to the curve $y = \sqrt{8x + 5}$ at the point where $x = \frac{1}{2}$ giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

[5]

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9. A solid circular cylinder has a base radius of *r* cm and a height of *h* cm. The cylinder has a volume of $1200\pi\ cm^3$ and a total surface area of *S* cm^2.

a. Show that $S = 2\pi r^2 + \frac{2400\pi}{r}$.

[3]

b. Given that h and r can vary, find the stationary value of S and determine its nature.

[5]

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10. (i) Differentiate $y = (3x^2 - 1)^{\frac{-1}{3}}$ with respect to x.

[2]

(ii) Find the approximate change in *y* as *x* increases from $\sqrt{3}$ to $\sqrt{3} + p$, where *p* is small.

[1]

(iii) Find the equation of the normal to the curve $y = (3x^2 - 1)^{\frac{-1}{3}}$ at the point where $x = \sqrt{3}$.

[3]

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11. At the point where $x = 1$ on the curve $y = \frac{k}{(x+1)^2}$, the normal has a gradient of $\frac{1}{3}$.

a. Find the value of the constant k.

[4]

b. Using your value of *k*, find the equation of the tangent to the curve at *x* = 2.

[3]